Modelling opinion dynamics realistically

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A realistic model of opinion dynamics

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Semester: VI

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Declaration: I affirm my faith that I have identified all my sources and that no part of my dissertation paper uses unacknowledged materials.

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Mentor Student

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Introduction

Once upon a time, most people who worked with numbers did not connect with the society, and most people who worked with people, were afraid of mathematics. It changed, and the subject of opinion dynamics was born. Opinion dynamics is the study of propagation of opinion through a population – how it spreads, recedes, or partitions the society. Men like Hobbes, Laplace, Comte, Stuart Mill, Majorana and many others led to the eventual formation of the subject [1,2,3]. The 19th century ideas were mostly philosophical, and only recently these ideas are becoming mathematical models. This has to do with the availability of large databases, and the emergence of networks like social media. They have brought new variables into the dynamics of a society, and have complicated modelling the behavior of an already inherently complex system. But they were instrumental in showing us that dynamic models can be constructed, and maybe, the problem isn't impossible to solve after all. Even where a closed-form mathematical solution is not available, Monte Carlo simulation on the basis of the model can provide considerable insight into what might happen in real life scenarios.

Opinion dynamics is different from a complex system of physics. In physics, the basic laws of the macroscopic world are known, and so, deterministic prediction of every macroscopic particle is possible. The complexity arises from the numerous interaction that take place between an immensely large number of particles, and in a large variety of physical conditions. In social dynamics, the basic elements (humans) are not well understood. Psychology is an approximate science, and specific rules that guide all humans haven't yet been found (hopefully they don't exist, for in the unknown there is beauty). This lack of knowledge regarding the basic elements of a society or network, along with their complex interactions make modelling a difficult (and therefore interesting) task. A popular way out of this problem is to model one aspect only. This empowers us with models that can work out the behavior of collectives without knowing exactly how each participant functions. They let us see how patterns arise out of apparently erratic and chaotic behavior of numerous individuals.

Existing work

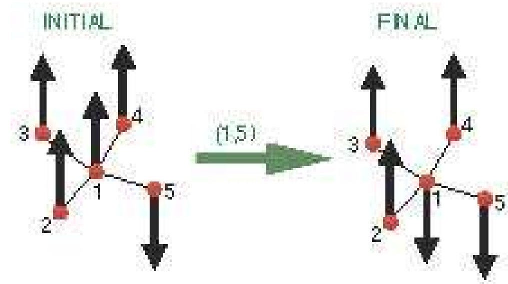
The basic Ising spin model

One of the most basic models of opinion dynamics is the Ising spin model. Here, the opinion of an individual is considered as a binary – spin up, or spin down. Spin couplings represent peer interactions and external information is the magnetic field. This model may appear too reductive, considering the complexity of a person and of each individual position. But often, life does offer people with a limited number of positions on a specific issue, sometimes indeed a binary: to take a subject or not, to choose a product or not, to buy or to sell, to stay with one group or another etc. The model is beautifully simple – it foresees a phase transition from an ordered to a disordered phase. It's intuitive, it's simple, but it's too simple to model a society faithfully. This has led to the development of many other models. This model is actually a special case of the majority rule system (explained below), with group size fixed at 2.

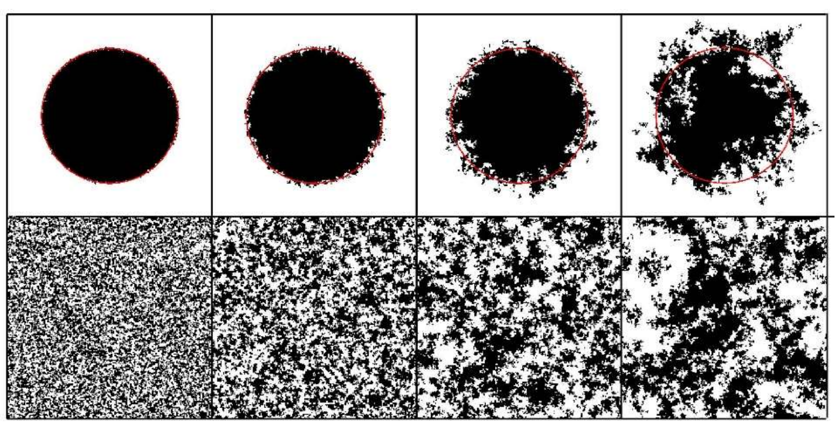
These models can be multidimensional, some have a continuous opinion spectrum, and each follow a different interaction, or opinion update rule.

The voter model

A very useful framework is the voter model. It was originally introduced to analyze competition of species [4]. The model has then been attracting a large amount of attention in the field of opinion dynamics, and its name stems from its application to electoral competitions [5]. In this model, each agent in the population holds one of two discrete opinions, $O = \pm 1$, similar to the Ising model mentioned above. Agents are connected by an underlying graph defining the topology of the system. At each time step, a random agent i is selected along with one of its neighbors j and the agent takes the opinion of the neighbor. Thus, while spins in the Ising model try to align with the majority of their neighbors, voter dynamics involve one neighbor only, hence the majority does not play a direct role, but is felt indirectly through peer interaction. A generalized framework that encompasses different variations of voter dynamics has been introduced recently in [6].



Suppose that in the dynamical evolution of the model, which considers an interaction between an agent and one of its neighbors chosen at random, the agent number 1 was selected in the configuration of the left part of the figure. With probability 3/4 it will remain with a positive opinion since three of its neighbors have a positive opinion (the agents 2, 3, and 4), while with probability 1/4 it will change it since one of its neighbors has a negative opinion (the agent 5). In the example, the final state on the right refers to this latter event.



The adjacent figure shows evolution of a two-dimensional voter model starting from a circle (top) or a fully disordered configuration (bottom). The white and black colors represent the positive and negative opinions respectively. From the top panel we can see how the black area remains practically constant during the dynamics and the original circular shape is destroyed. In physics, this signals a lack of surface tension [6].

The voter model in two dimension, with temperature, has been applied to explain opinion change in financial markets [7]. The temperature (a type of noise) is associated to the nervousness of agents (fear). Through a feedback between the status of the entire agent population (market imbalance) and the temperature, nervousness becomes an evolving feature of the system. This passes through two types of metastable states, either long-lived striped configurations or shorter mean-field like states.

The update formula for the basic voter model expresses the opinion of the ith individual at time $t + 1$ as

$$O_{t+1}(i) = O_t(j),$$

where the j is selected from the index set N_i of the neighbors of i with equal probability.

The Majority Rule model (MR)

A different approach is the majority rule model. The MR model was first proposed to describe public debates [8]. Agents take discrete opinions ± 1 and can interact with all other agents (complete graph). At each time step, a group of r agents is selected randomly and they all take the majority opinion within the group. The group size can be fixed or taken at each time step from a specific distribution. If r is odd, then the majority opinion is always defined, however if r is even there could be tied situations. To select a prevailing opinion in this case, one possibility is to introduce a bias in favor of a particular opinion, say $+1$, or the current opinion of the individual. This idea is inspired by the concept of social inertia [9]. The MR model with opinion bias was originally applied to describe hierarchical voting in society [10,11,12,13] with the discussion recently extended to three discrete choices for hierarchical voting [14]. Recent extensions have been used to explain results of public debates on different issues such as global warming, evolution theory, H1N1 pandemic [15]. These include two types of agents, floater and inflexible, where inflexible agents do not change their opinion. It is shown that, for the case where not enough scientific data is available, the inflexible agents are those that drive the result of the debate. Hence, a strategy for winning a debate is the acquisition of as many inflexible agents as possible. Also, the analyses indicate that a fair discourse in a public debate will most likely lead to losing, while exaggerated claims are very useful for winning. Similar results are presented in [16], where contrarians, i.e. agents who take the minority opinion of a group, are also introduced. In a different variation of the model [17], collective beliefs are introduced as an individual bias to select one or the other opinion, in case of a tie in voting. Here only pair interactions are analyzed. The study shows that collective beliefs are very important in determining the results of the debate, and again, a winning strategy is acquiring inflexible agents, which may mean using overstated or exaggerated statements.

The update formula for the majority rule model postulates that the opinion of individual i at time $t + 1$ is

$$O_{t+1}(i) = \text{sign}\left(\sum_{j \in N_i} O_t(j)\right),$$

where N_i is the index set of neighbors of i. Tie, if any, is broken through a pre-determined bias towards one opinion.

Considerations for new modelling

The models I have studied seemed to ignore the fact that people have different personalities. Also, not every personality is present in the same numbers. To represent this diversity, I have prepared a *continuous* scale of personalities over the interval 0 to 1. At the left end of the scale, are people who are ready to change ('floaters' in the parlance of the foregoing discussion). At the middle are lazy people. At the right extreme are people who want to persuade others to change, but not change themselves (those described as 'inflexible' in the above discussion). The personality score is assigned at random from a beta distribution. The shape parameters (α, β) of the beta distribution control the mix of different types of people present in the society.
The initial opinion (opinion is considered as a binary: ($+1$ or -1) of each member is assigned randomly, the fraction of people with one opinion is controlled by a specified probability of opinion $+1$.
In the models reviewed above, for any given individual all the other individuals have status neighbor or non-neighbor. Only the opinions of the neighbors matter in shaping the evolution of the opinion of the chosen individual. In reality though, the weight of opinions of others is not a binary variable. Some "friends" matter more than others. In order to model this reality, I assign the weight of a "friend" on a particular individual as a decreasing function of the proximity of the "friend" measured in some scale. The weights are designed to sum up to one.
I have prepared two models, one is an offspring of the voter model, the other of majority rule. They differ only in the rule dictating how an agent changes his or her opinion when they interact with another agent, and whether they change their opinion at all or hold on to their previous line of thought.

The significant one model

In my version of the voter model, an individual has the highest probability of communication with the closest friend – the person with the highest weight. The outcome of the interaction depends on the personalities of the two people. If the neighbor chosen for interaction harbors an opposing opinion and a higher personality score, there is a high chance of an opinion flip. In order to emulate free will in the real world, I added a random noise term to the interaction.

The update formula for the proposed voter model is

$$O_{t+1}(i) = \text{sign}\big(O_t(j)p(j) + O_t(i)p(i) + Z_{t+1}(i)\big),$$

where $Z_{t+1}(i)$ is a random noise term, $p(j)$ is the personality score of j and j is selected from the index set of individuals except for i, the probability of its selection being

$$w(i,j) = \frac{e^{-\frac{(x_j-x_i)^2+(y_j-y_i)^2}{2h^2}}}{\sum_k e^{-\frac{(x_k-x_i)^2+(y_k-y_i)^2}{2h^2}}},$$

and the sum in the denominator is over *all* agents except i. Here, the pair (x_i, y_i) represents the locational coordinates of the ith individual. This location need not be geographical. Rather, it should be determined by the topology that best describes the interactions. For example, in the special case of a web-based social network, the distances can be expressed by number of 'friends that separate two individuals. It may be recalled that a specified distance matrix can always be approximately described by inventing a matching coordinate system by the technique of multi-dimensional scaling. In fact, the specified distances can also directly be used in place of the Euclidean distance used in the formula of $w(i,j)$. The parameter h represents the effective size of the neighborhood. A large value of h leads to an individual being influenced by a large number of 'friends'.

The significant others model

In my majority rule based model, a person interacts with everyone else. The highest weight, or importance is given to the closest friend, and this weight diminishes with increasing distance of the neighbor. Though the outcome depends on the personalities and opinions of all the people in the world, effectively a few close friends matter, because of the importance or weight given to them. Here too the outcome depends on the personality of the agent in question, and a random noise working as free will.

The update formula for the proposed majority rule model is

$$O_{t+1}(i) = \text{sign}\left(\sum_j w(i,j)O_t(j)p(j) + O_t(i)p(i) + Z_{t+1}(i)\right),$$

where $Z_{t+1}(i)$ is a random noise term, $p(j)$ is the personality score of j,

$$w(i,j) = \frac{e^{-\frac{(x_j-x_i)^2+(y_j-y_i)^2}{2h^2}}}{\sum_k e^{-\frac{(x_k-x_i)^2+(y_k-y_i)^2}{2h^2}}},$$

and both the sums are over *all* agents except i.

Experiments and observations:

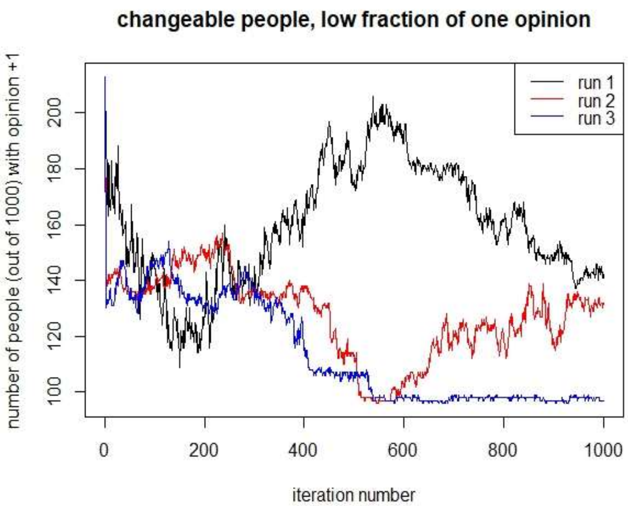
The key to unlocking secrets

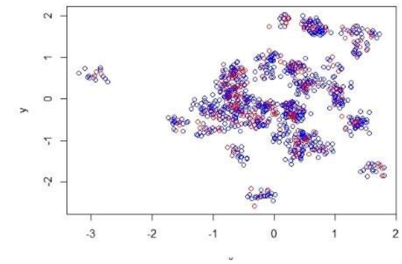
Progression of opinion in various social scenarios have been studied with the two models. Results from the significant one model are displayed on the top, results from the significant others model at the bottom. The plot on the left shows the count of people with opinion $+1$ vs time t. The upper right scatter shows the initial opinion of agents, and the bottom right one the final opinions. Red signifies the $+1$ opinion, blue shows -1. The population size is taken as 1000.

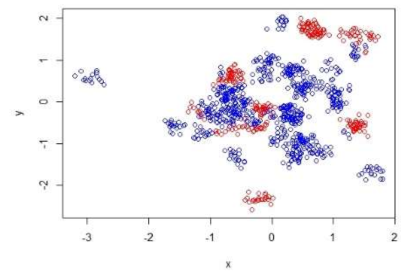
The experiment is carried out for different choices of parameters of the beta distribution of personality scores, different choices initial probability of $+1$ and the choice between the voter or majority rule model. However, the neighborhood size h is fixed at a size that corresponds to just four 'friends' accounting for 90% of the weights $w(i, j)$. This is because an average person is known to have a close group (of 'friends') with size 4.

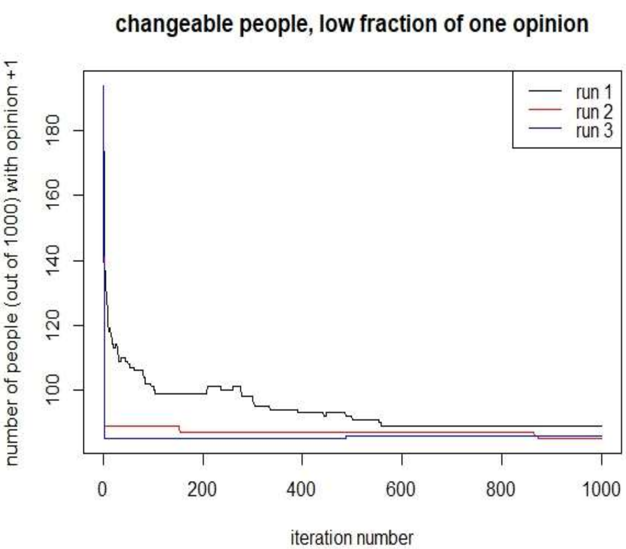
The next few pages show the output in different cases with interpretation.

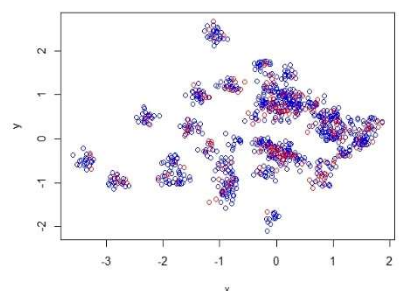
A population with a high number of changeable people, and a low percentage of people having opinion +1 ($\alpha = 8,\ \beta = 2$), (20%)

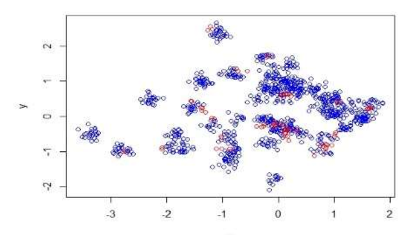






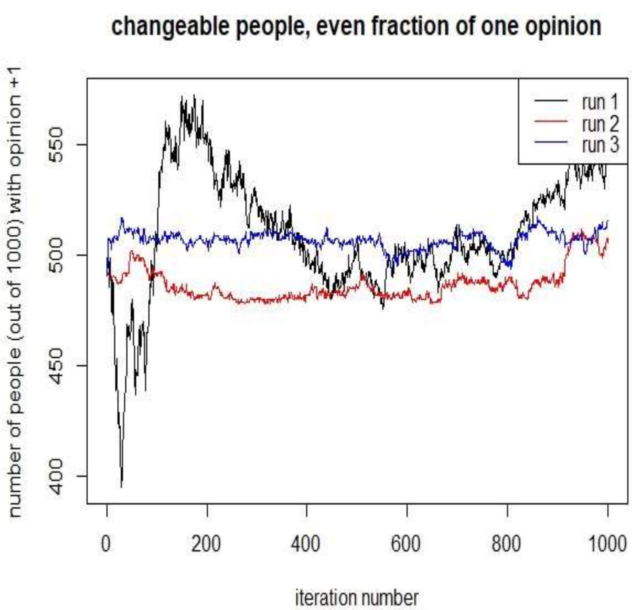


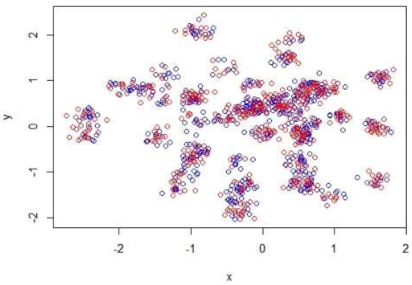


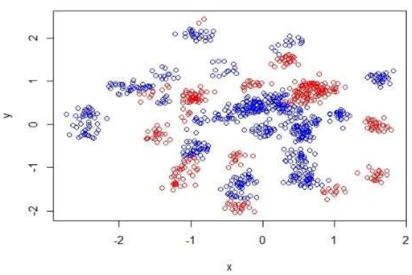


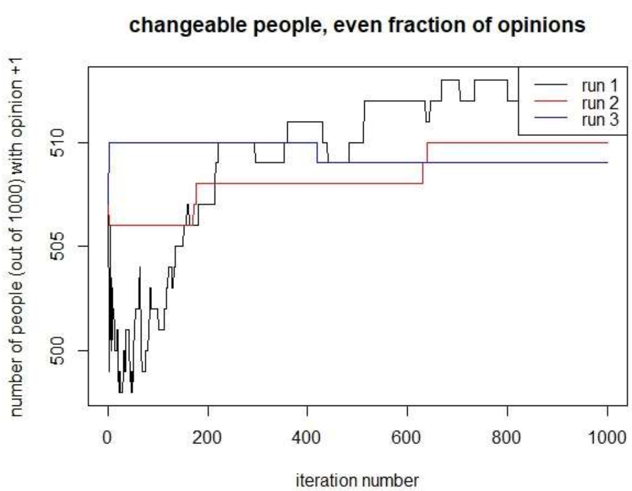
In this situtation, one on one communication leads to survival of minority opinions in self sustaining clusters. Group communications show a steady decline, and then saturation at some very low supporter number.

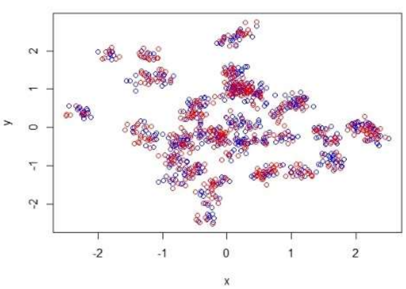
A population with a high number of changeable people, and an equal percentage of people having opinion +1 ($\alpha = 8,\ \beta = 2$), (50%)

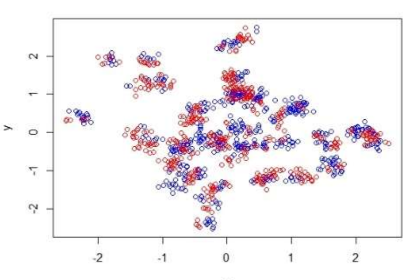






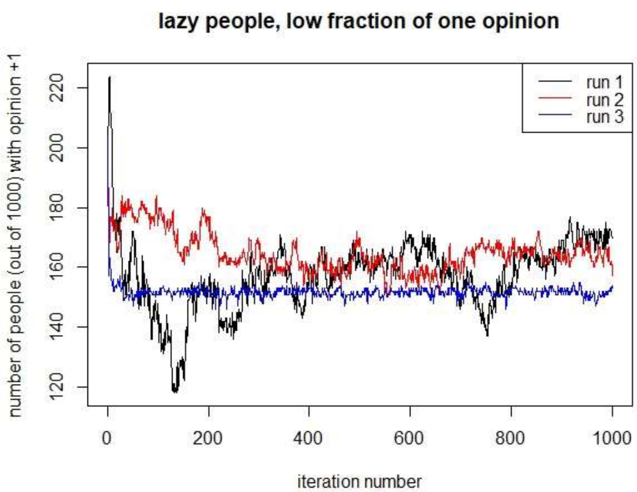


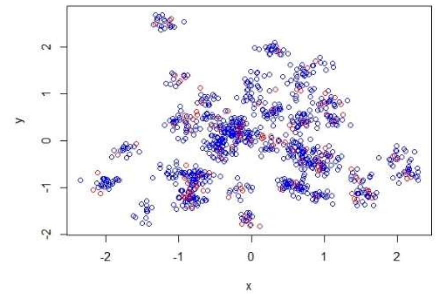


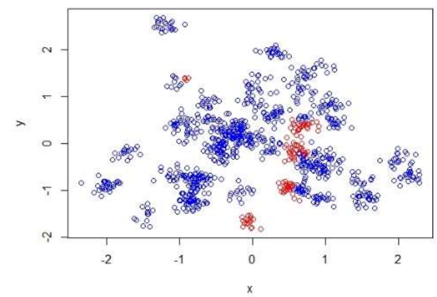


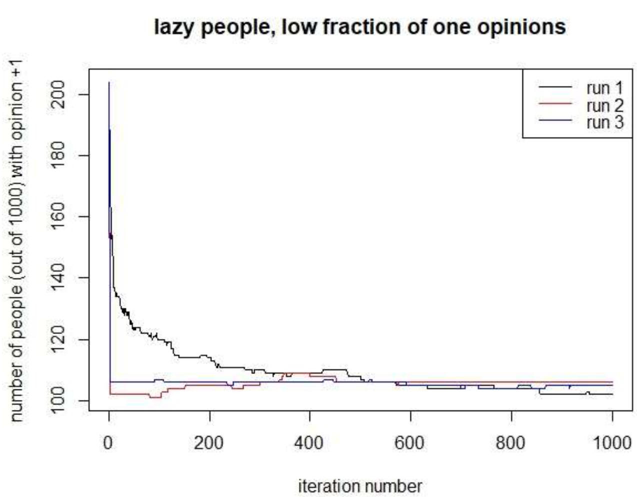
Here results from both modes of communication are somewhat similar. The significant other model predicts population fluctuations around some number, which changes over time somewhat. The significant others model proposes a more peaceful convergence into a quite narrow population range. Both models show sudden larger fluctuations to settle down in the final range.

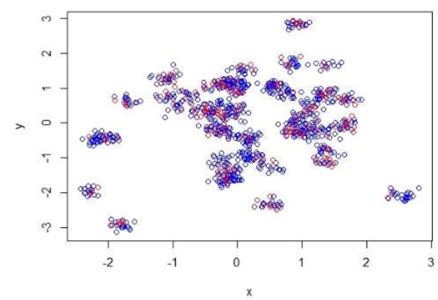
A population with lazy people, and a low percentage of people having opinion +1 ($\alpha = 5$, $\beta = 5$), (20%)

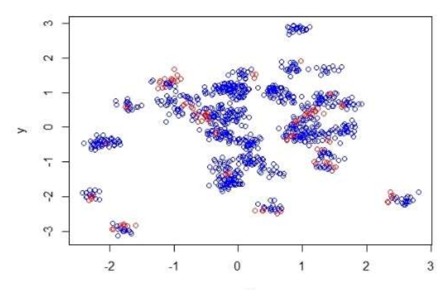






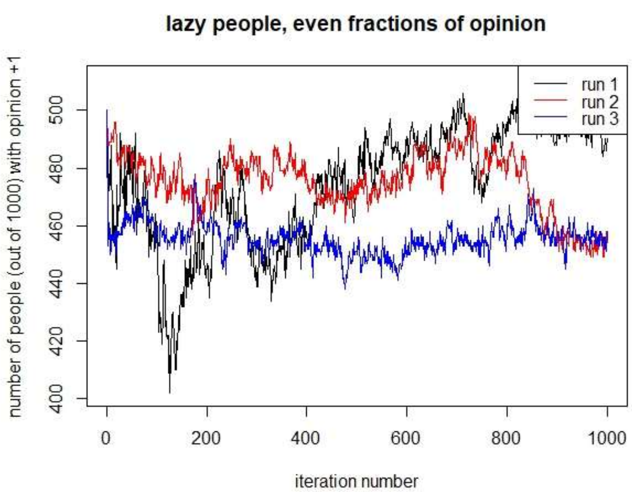


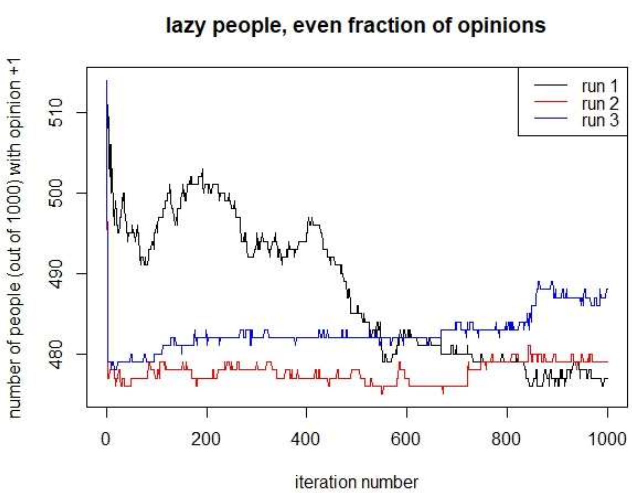


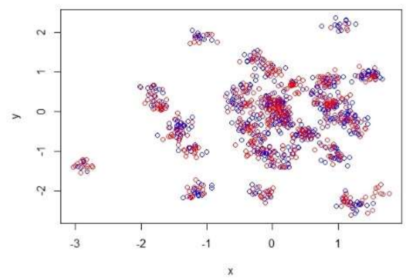


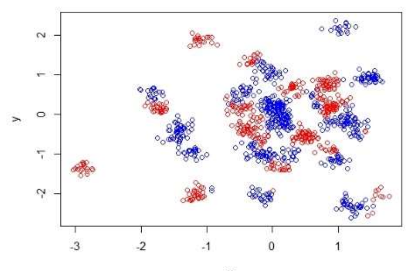
The majority dictated opinion shifts lead to almost no propagation in these conditions. Peer to peer communication is a bit more ambitious, the clusters do spread out, but only to recede, and repeat. Over short time scales oscillations are present, but they are too weak in both cases to result in any major propagation of a particular opinion.

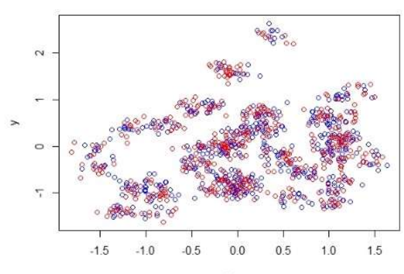
population with lazy people, and an even percentage of people having opinion +1 ($\alpha = 5,\ \beta = 5$), (50%)

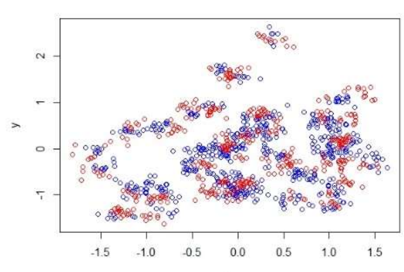






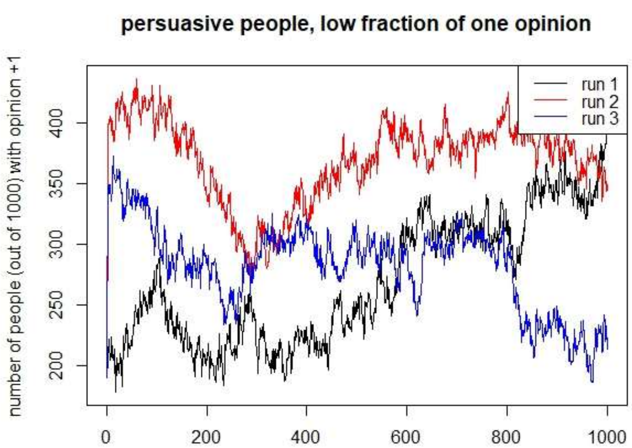


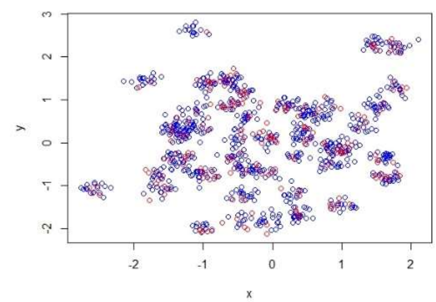


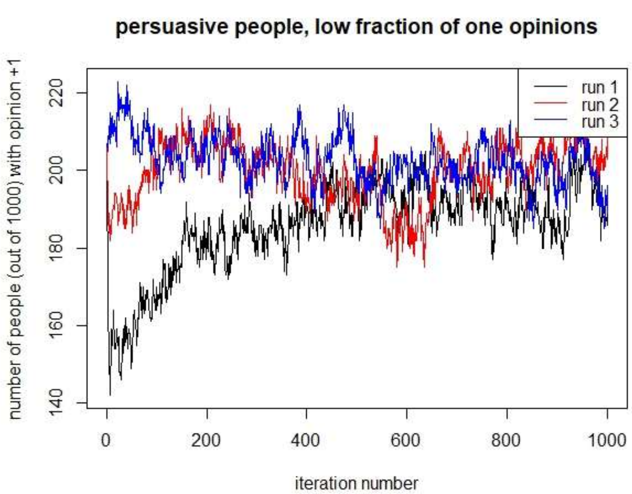


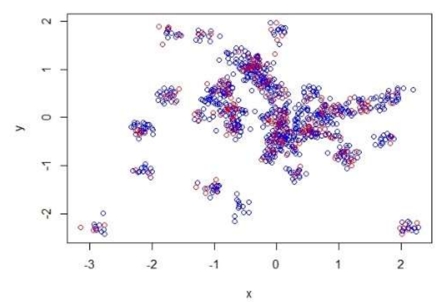
In the depicted society, both models are eager (the majority model more noticeably) to converge to some point. This point does change over very long timescales.

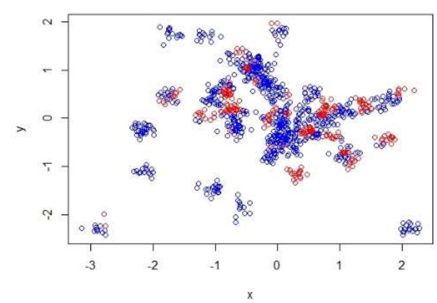
population with persuasive people, and a low percentage of people having opinion +1 ($\alpha = 2$, $\beta = 8$), (20%)



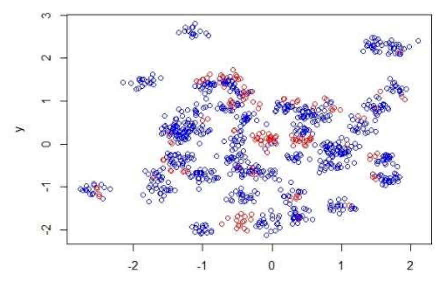




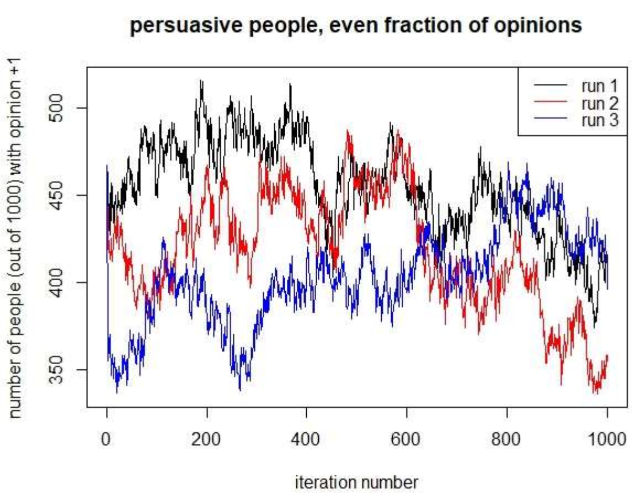


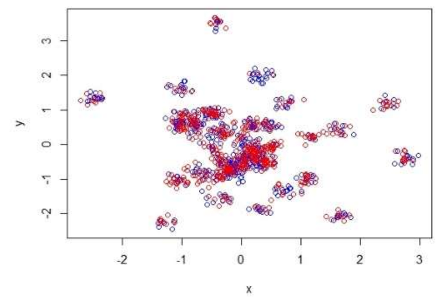


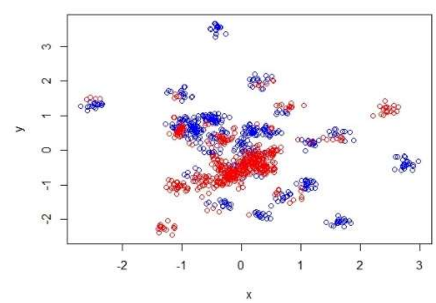
One on one communications here lead to a tussle between the two pinions, with neither getting the upper-hand for any considerable time. Group communications lead to a saturation point, almost independent of initial conditions. This because of formation of clusters, outcast from the majority opinion.

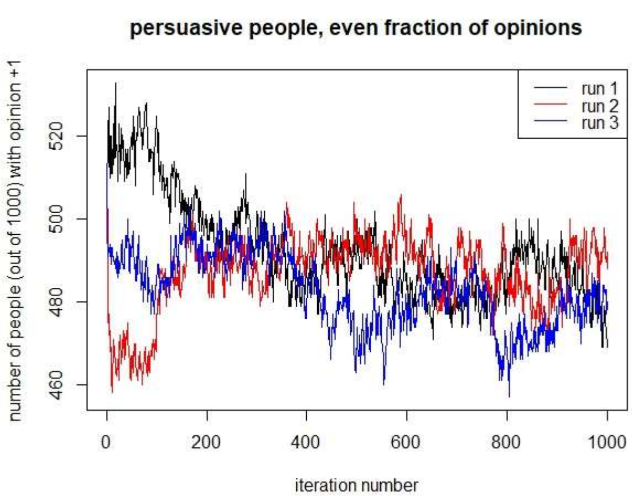


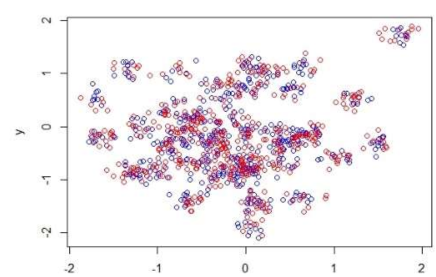
population with persuasive people, and a low percentage of people having opinion +1 ($\alpha = 2$, $\beta = 8$), (20%)

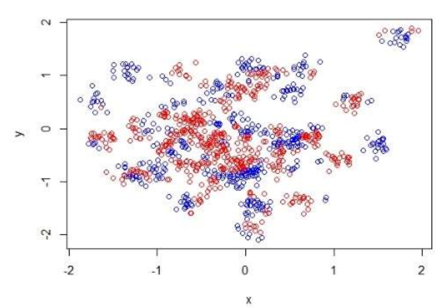








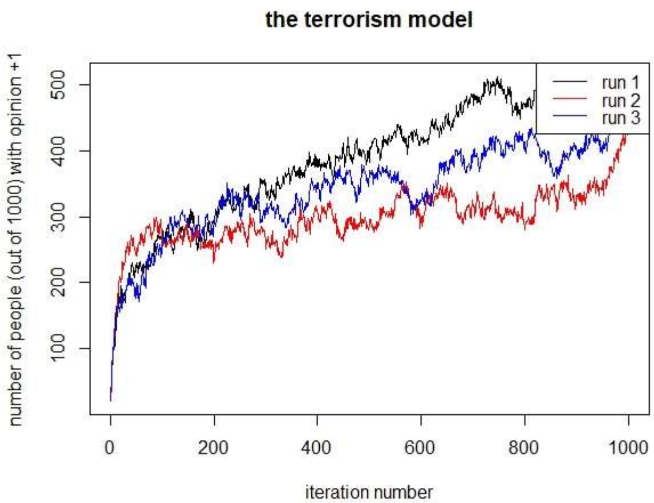


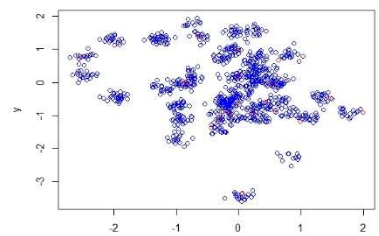


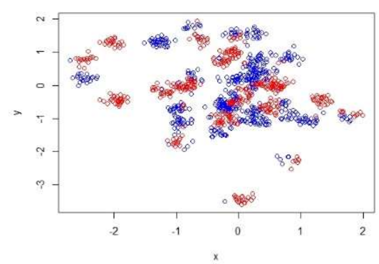
In this type of society, the division is most visible. Agents remain separated, but strong willed people manage to create clusters around themselves. Clustering is faster with group communications here. One on one communications lead to formation of more intense groups.

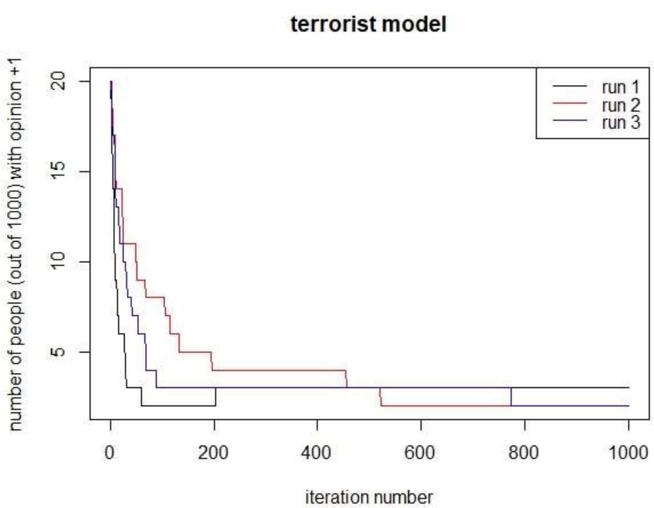
lessons learnt: towards a prediction and a test of the model

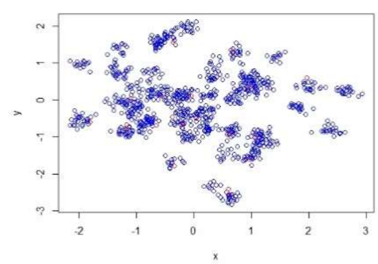
A scenario was simulated where very few but very persuasive people were assigned the opinion +1 initially. The following plots are accounts of what happened.

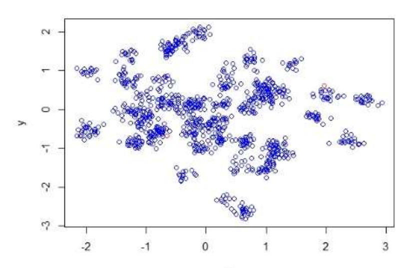












If a lot of ready to change people are waiting, and very few people are extremely persuasive, and hold a minor opinion, it can lead to two outcomes: rapid propagation of the minority opinion till it becomes the new majority, or the minority opinion gets almost extinguished. The first possibility is realized only in the significant one model interaction system – one on one communication. The second possibility plays out when the communication is in the significant others style – one person listening to a group.

These observations have deep implications. They mean that if any change is to be made, the only way is one on one communication. Even minorities can become majorities if the proponents are persuasive and zealous enough.

In conclusion…

The models developed here seems to be good ones, reproducing social dynamics faithfully. They teach us the role of group communications in calming down severe issues, and the importance of perseverance in spreading awareness. I feel that these models have shown that despite the advantages of wide connectivity, one on one interactions with close friends is probably the best way to work, if diversity is to be saved. There are dangers of runaway fragmentation, and there are dangers of stagnancy, but maybe, armed with these models, we can anticipate the dangers and guide ourselves for better changes.

Further scope

1. There is a huge difference between opinion and action. Using opinion dynamics as a stepping stone, there is a possibility of building action dynamics. This will be a more comprehensive (but still incomplete) model of the world we live in.
2. The models presented here rely on a personality score. It would be an interesting task to develop an accurate method to translate a person to a score.
3. Fine tuning always remains to be done.

Acknowledgements

1. Dr. Rahul Sharma, my mentor in this project. He opened my eyes to this subject, and trusted me to work on it despite all indications that I shouldn't be trusted.
2. Dr. Debasis Sengupta. He helped me out with mathematical difficulties that plague me.
3. Dr. Sarbani Palit for providing me a machine to work on.
4. Supradipta Ghosal. She helped me make sense out of chaos. Also, she claimed no one works harder than Multimedia students, and inspired me to try to prove her wrong. She was right.
5. Supratim Dhar. He kept me awake.
6. Megha Murty. She refused to sing.

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